Representational Fluency in Learning and Problem Solving in Physics

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What is Representational Fluency?

"The ability to comprehend the equivalence of different modes of representation" (Sigel & Cocking, 1977)

"Comprehend Equivalence":

- Read out info presented in different representations.
- Transform information from one representation to other.
- Learn in one representation and apply to other.
- Others…

"Modes of Representation":

- Verbal vs. Mathematical
- Graphical vs. Equational
- Macroscopic vs. Microscopic
- Physical vs. Virtual
- Others…

Representational Fluency involves Transfer

Some Views of Transfer

- Identical elements must exist between situations.
- Knowledge must be encoded in a coherent model.
- Students either transfer or they don’t.
- Researchers/educators pre-decide what must transfer.
- Static one-shot assessment e.g. tests and exams.
- Focus mainly on students’ internal knowledge.

Transfer is rare.

E.g. Gick & Holyoak (1980), Reed & Ernst (1974), Thorndike (1906)

Some Emerging Views of Transfer

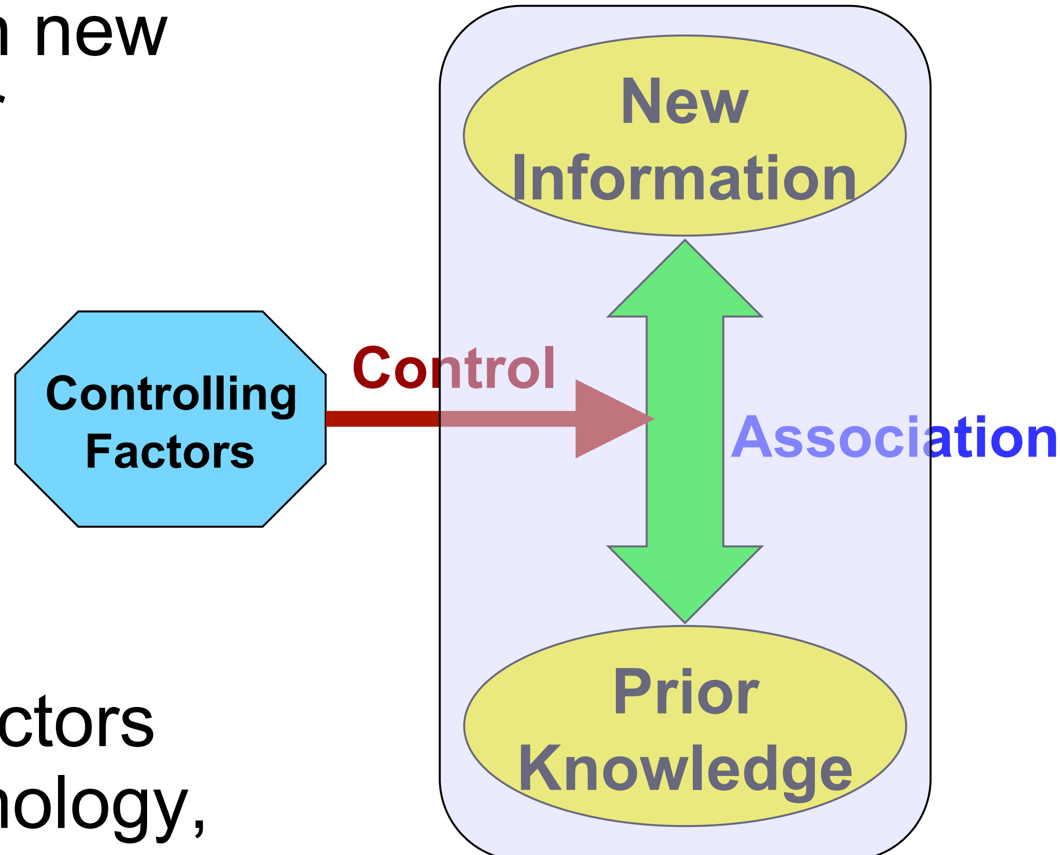
- (Re) construct knowledge in new context.
- Knowledge can transfer in pieces.
- Learners may transfer some pieces, but not others.
- We must examine anything that transfers.
- Dynamic, real-time assessment e.g. interviews.
- Focus also on mediating factors e.g. motivation.

Transfer is ubiquitous.

Hammer *et al* (2005), diSessa & Wagner (2005);
Bransford *et al* (1999), Lobato (2003, 1996), Greeno *et al* (1993)

Our View of Transfer

Transfer is the creation of associations between new information and prior knowledge.

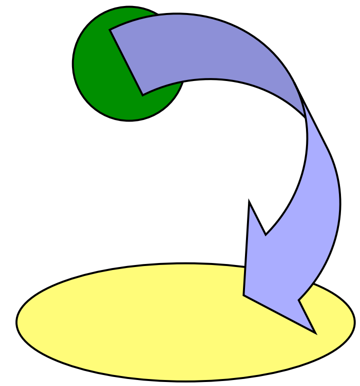


The association is controlled by other factors e.g. learners' epistemology, motivation, emotions, etc.

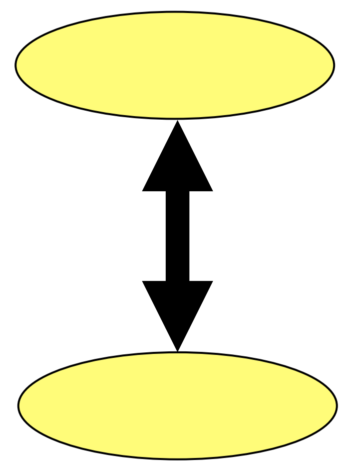
Redish (2004)

Two Kinds of Associations

- Assigning a new case to an existing knowledge element.
  - e.g. The electric field between two parallel plates is constant.



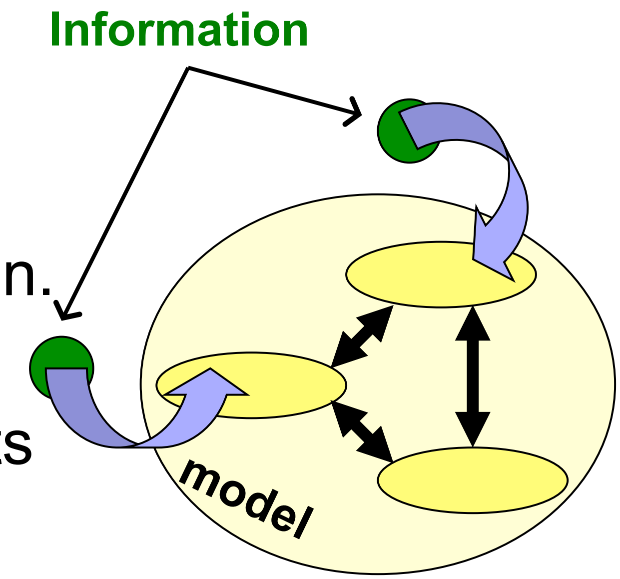
- Constructing an association between two knowledge elements.
  - e.g. Integral of Electric field is the Electric potential.



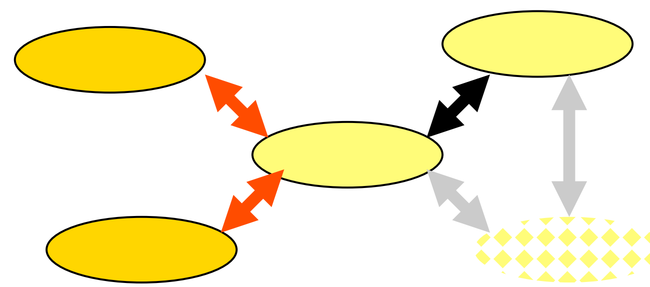
Two Kinds of Transfer

- ‘Horizontal’
  - Activating and mapping a pre-constructed model to a new situation.
  - Associations between read-out information of a situation & elements of model.

A “model” is a pre-created set of associated elements

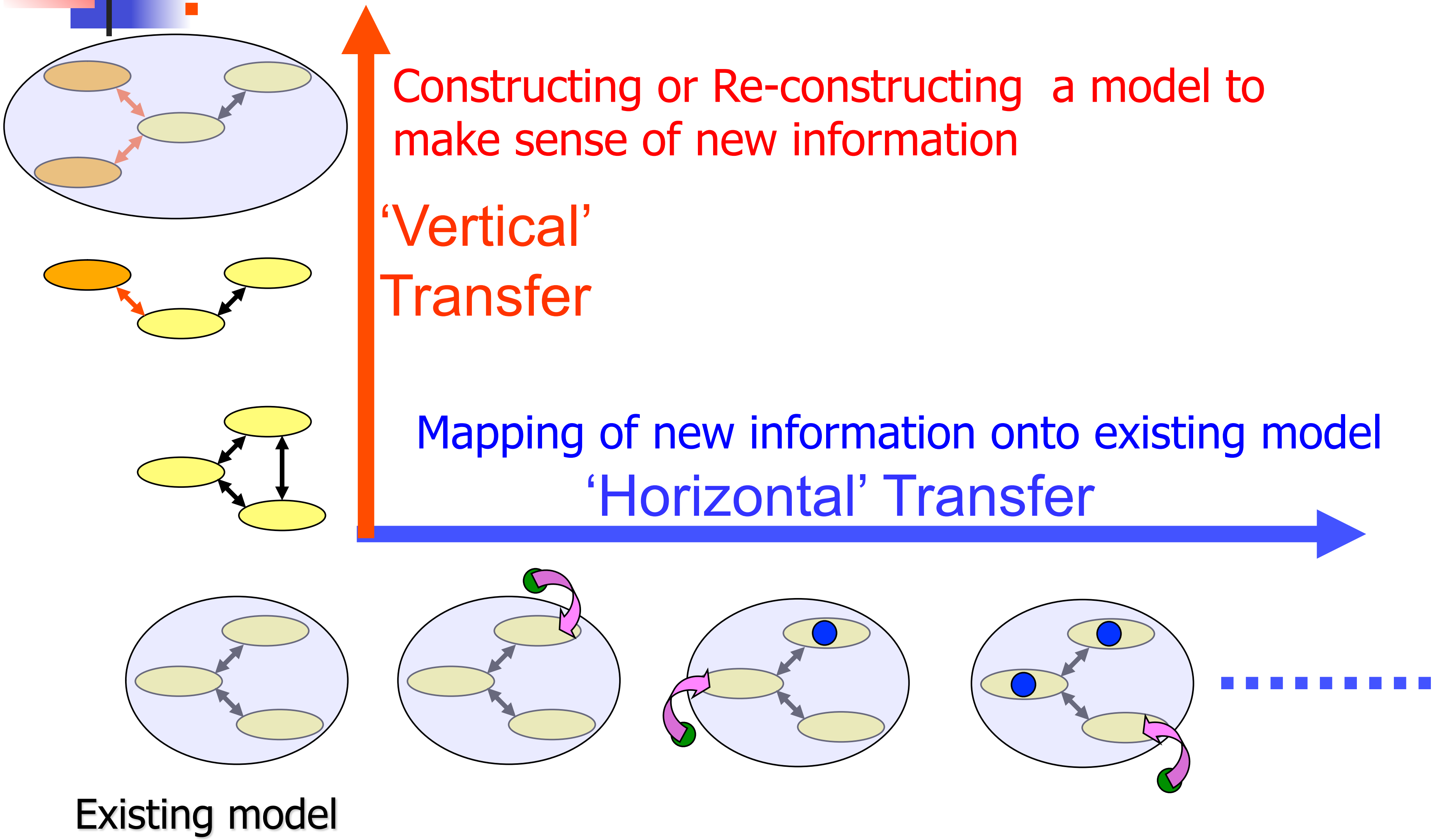


- ‘Vertical’
  - Constructing a new model to make sense of a situation.
  - Association between knowledge elements to create model.



New knowledge elements incorporated in model, others are discarded

Our Framework of Transfer



Alignment with Others' Views

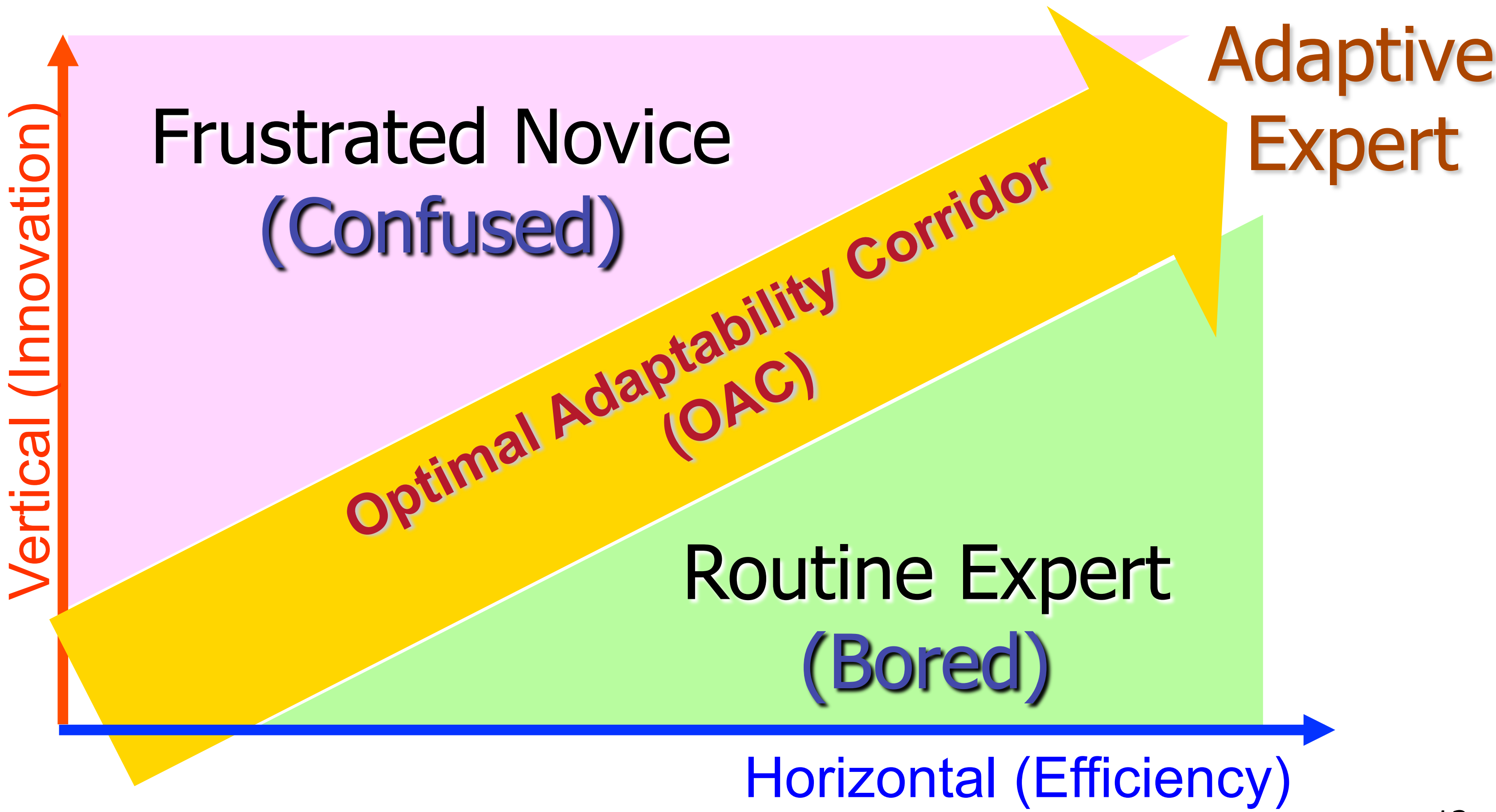
Horizontal	Vertical
Assimilation	Accommodation [1]
Efficiency	Innovation [2]
Model Development	Model Deployment [3]
Class C Transfer	Class A Transfer [4]
Low Road Transfer	High Road Transfer [5]
Applicative knowledge	Interpretive knowledge [6]
Sequestered Problem Solving	Preparation for Future Learning [7]
Used in structured, traditional contexts, which involves few internal representations activated repeatedly	Used in ill-structured, non-traditional contexts, which involves choosing, or constructing multiple internal representations [8]

[1] Piaget (1952) [2] Schwartz, Bransford & Sears (2005) [3] Hestenes (1987)
[4] diSessa & Wagner (2005) [5] Salomon & Perkins (1989) [6] Broudy (1977)
[7] Bransford & Schwartz (1999) [8] Jonassen (2003)

What Transfer do We Want?

Horizontal (Efficiency) **AND** Vertical (Innovation)

Striking a Balance: 'Optimal Adaptability Corridor'[1]



[1] Schwartz, Bransford & Sears (2005)

[2] Murray & Arroyo (2002)

Some Caveats

'Horizontal' & 'Vertical' Transfer…

- are not mutually exclusive.
  - A given thinking process might involve elements of both 'horizontal' and 'vertical' transfer.
- cannot be universally labeled.
  - What is perceived as 'vertical' transfer by a novice may be perceived as 'horizontal' transfer by an expert.

Possible Research Questions (RQs)

- How do students engage in ‘horizontal’ and ‘vertical’ transfer?
- Under what conditions do they engage in each?
- Is there a preferred sequence for these processes?

⋮

and several others….

RQs For this Talk…

How does the sequence in which learners interact with different representations affect

- learning?
  - Study 1: Learning using Physical vs. Virtual Representations
- problem solving?
  - Study 2: Solving Problems in Numerical vs. Graphical vs. Equational Representations

Study 1: Background

- Previous studies -- mixed results
  - Virtual outperform analogous physical experiments
    - Zacharia, Olympiou, & Papaevripidou, 2008
    - Finkelstein, et al., 2005
  - No difference in learning : physical vs. virtual
    - Klahr, Triona, & Williams, 2007
    - Zacharia & Constantinou, 2008
- Zacharia & Constantinou (2008)
  - More research is needed to describe how physical and virtual manipulatives should be integrated in a curriculum.

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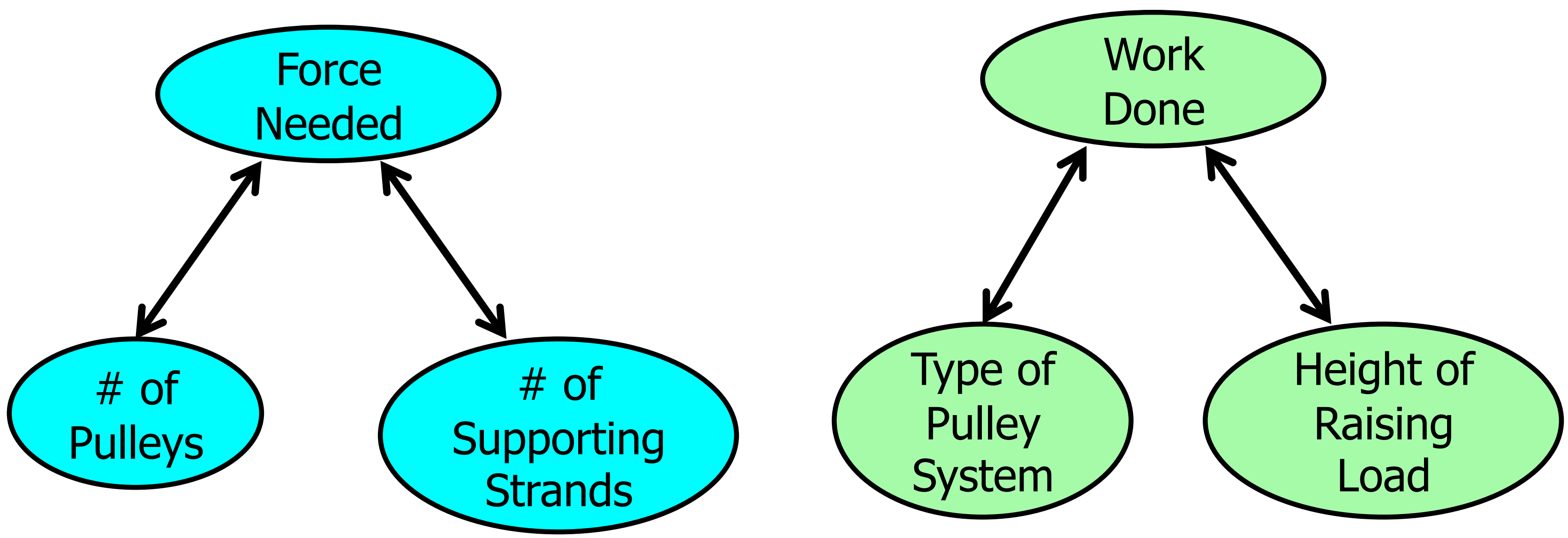
Study 1: Research Questions

When students use both physical & virtual representation…

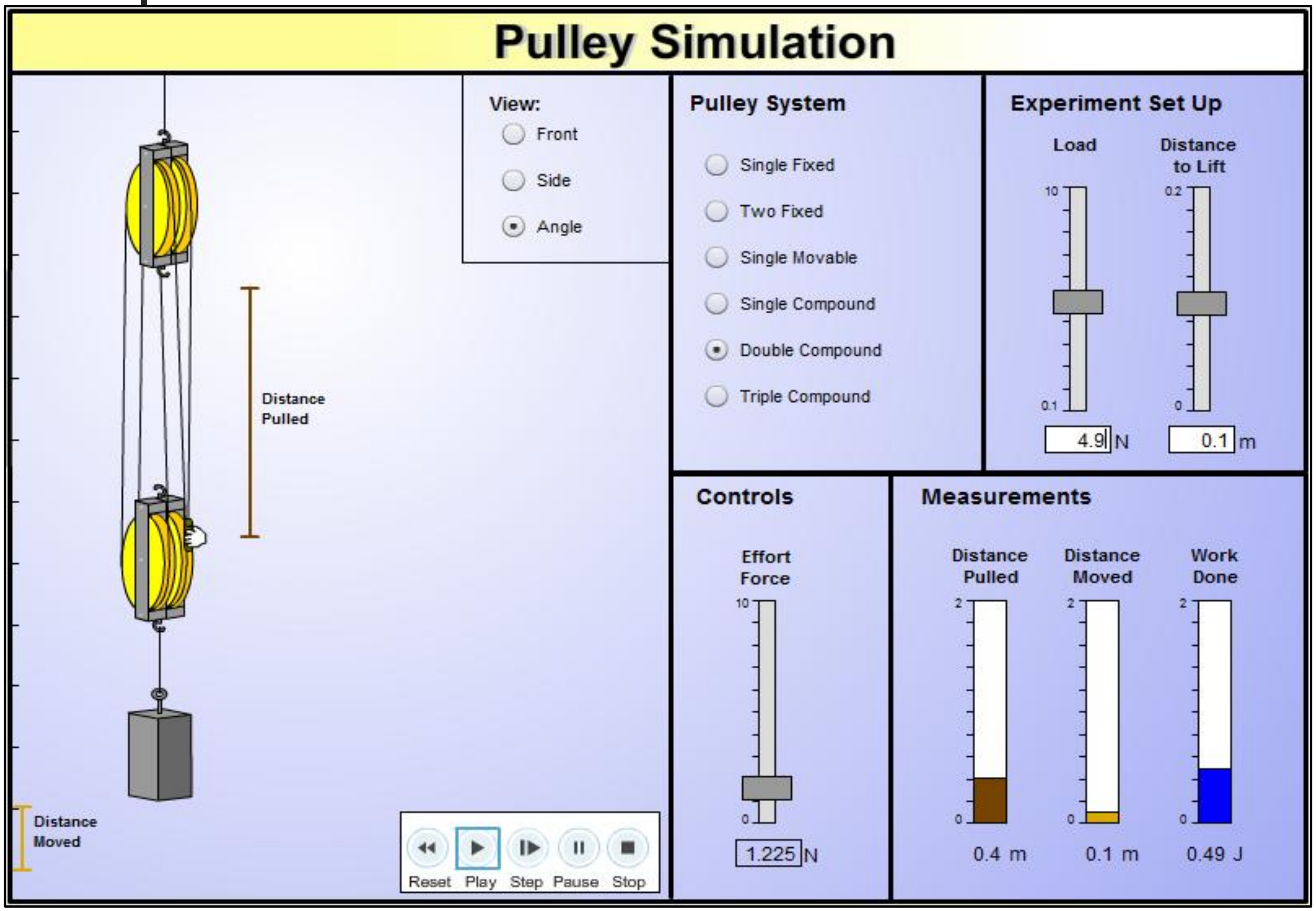
- How does their learning from the two representations compare?
- How does the sequence of using the physical and virtual representations affect students' learning?

Study 1: Research Context

- CoMPASS Curriculum (Puntambekar et al, 2003)
  - Concept Mapped Project-based Activity Scaffolding System
  - Integrates: Hypertext + Activities (Physical/Virtual)
- Pulley Unit : Two-hour lab
  - Targeted models:



Study 1: Physical & Virtual Representations



Virtual

Physical

Study 1: Research Design

PV Sequence (N=61) | **VP Sequence (N=71)**

Physical-Virtual Sequence | **Virtual-Physical Sequence**

Pre-Test

- 13 multiple-choice conceptual questions
- Cornbach Reliability ~ 0.75

Physical Experim

Mid-Test

st-

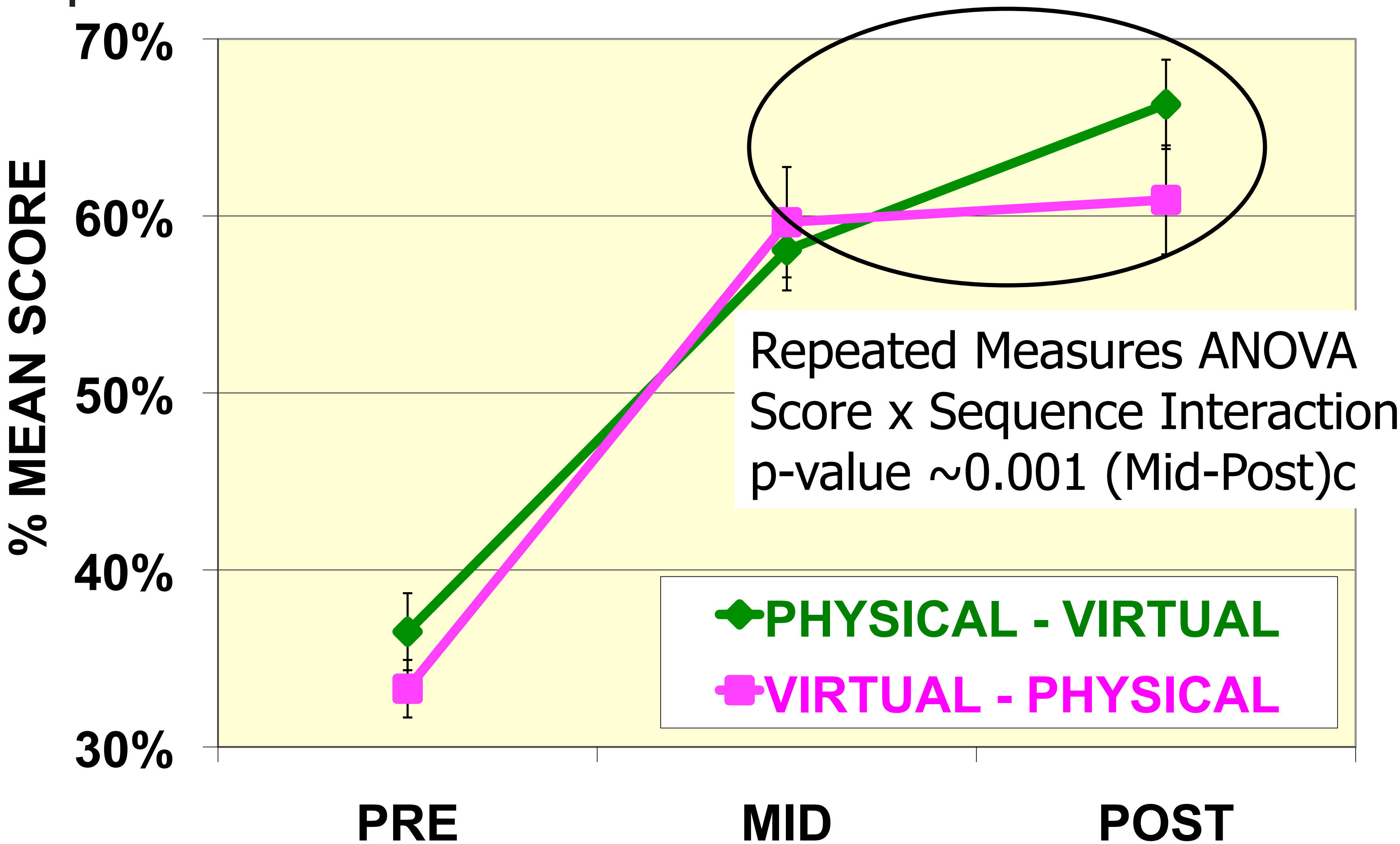
Physical-Virtual Sequence:

- Make predictions
- Set up various pulley systems
- For each …
  - Measure Force needed
  - Measure Distance pulled
  - Calculate Work & PE
- Discuss trends across systems

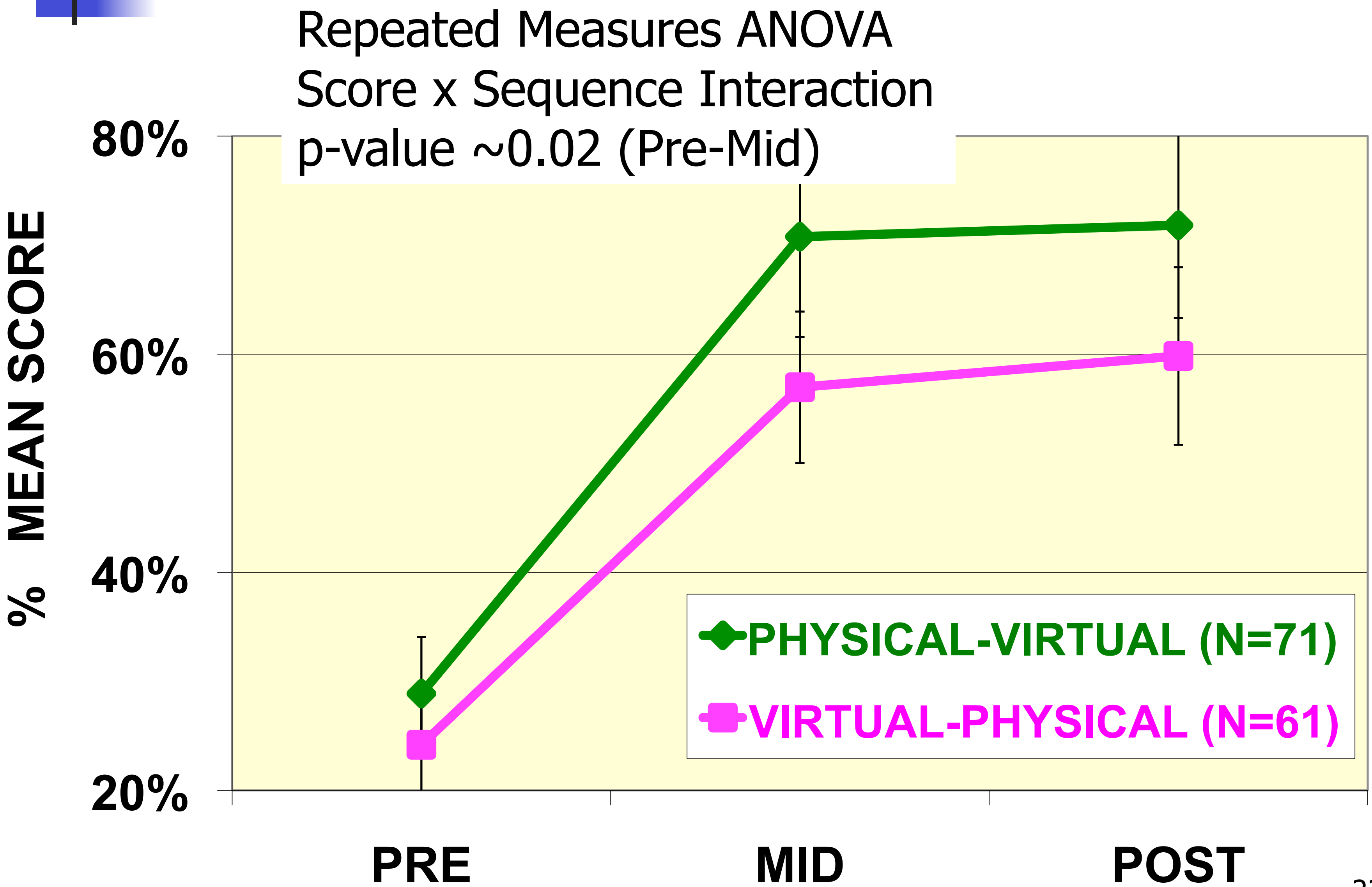
Virtual-Physical Sequence:

- Make predictions
- Choose various pulley systems
- For each …
  - Observe Force needed
  - Observe Distance pulled
  - Observe Work & PE
- Discuss trends across systems

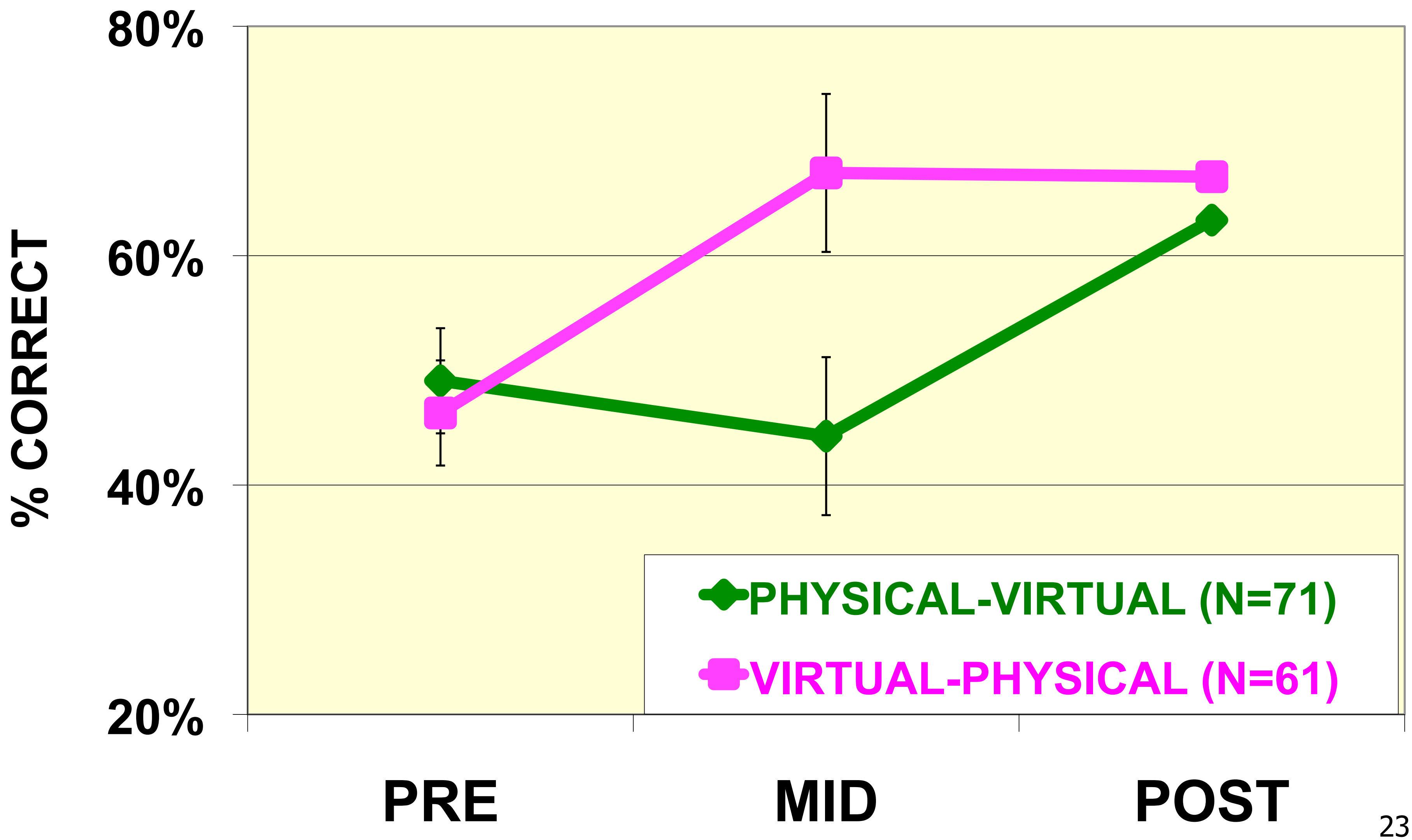
Study 1: Overall Test Performance



Study 1: 'Force' Questions on Test



Study 1: 'Work' Questions on Test



Study 1: Why these Results?

Two possible effects: Differential

- Cue salience?

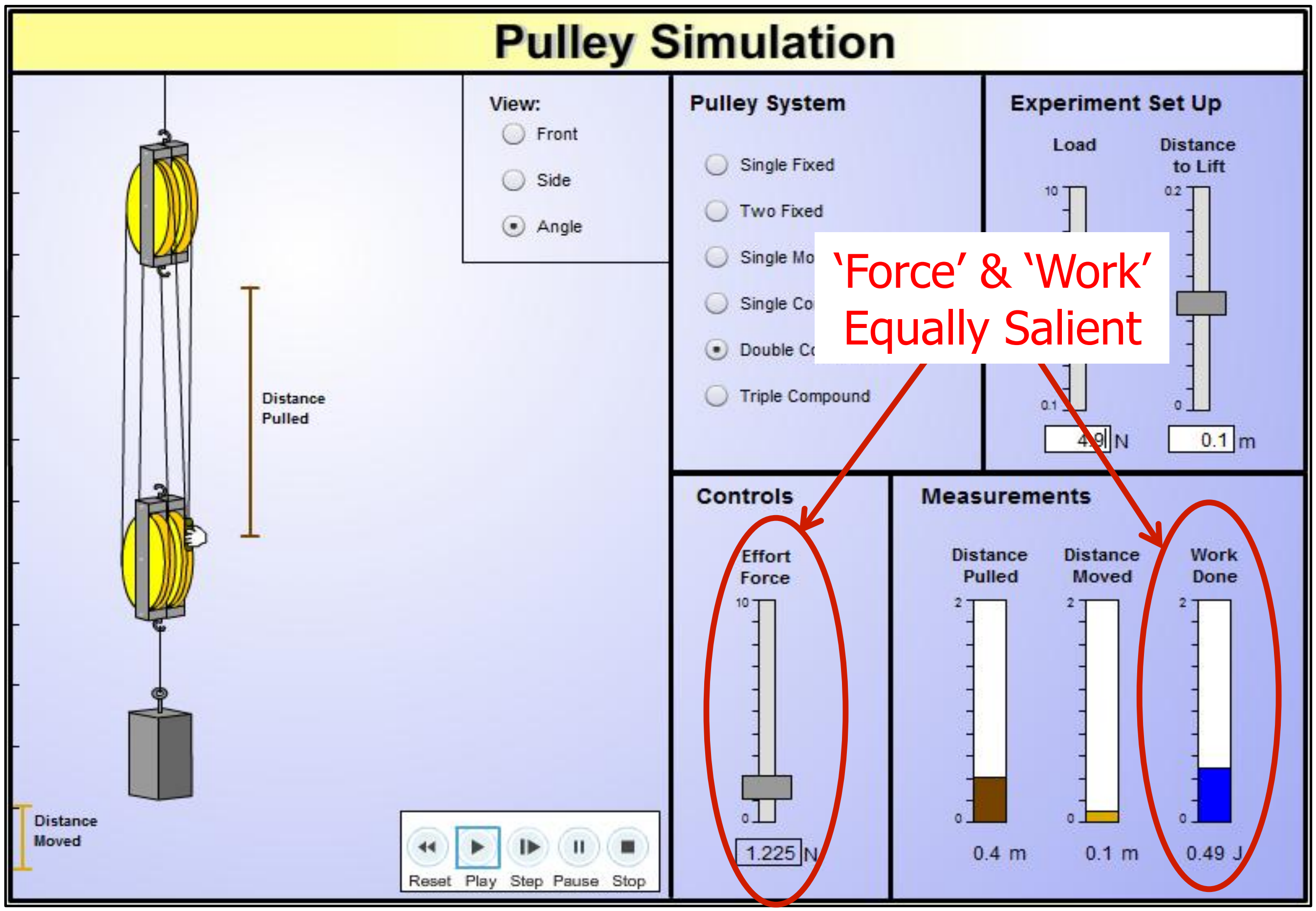
  (Denton & Kruschke, 2006)

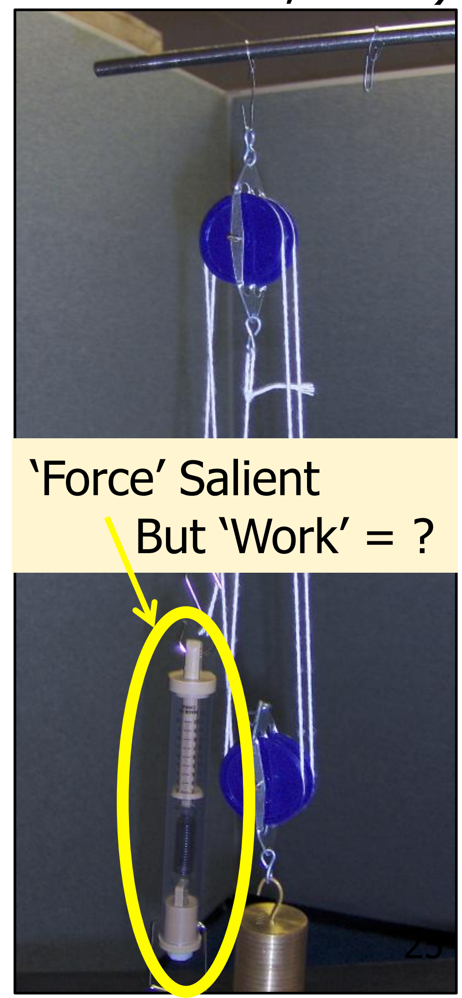
- Ambiguous Data?

  (Chinn & Brewer, 1993)

Study 1: What Causes Differential Cue Salience?

- Superiority / Noticing effect? (Lindgren & Schwartz, 2009)





Study 1: Implication of Differential Cue Salience

- Overshadowing? (e.g. Heckler, et al 2006)

	Physical P	Virtual V
'Force' Cues	High	High
'Work' Cues	Low	High

	Virtual V	Physical P
'Force' Cues	High	High
'Work' Cues	High	Low

Increasing Salience: Learning

Salience is high in both: Learning from whichever cue is presented first: Primacy effect

Decreasing Salience: Overshadowing

Study 1: Ambiguous Data

- Data that is learner (Chinn & Brewer, 1993)...
  - ambiguous may be ignored by the learner
  - Unambiguous may facilitate learning
- Ambiguity due to: measurement error, friction, etc.
- In our case, for student data on 'Work'

Type of Pulley System	Work value determined in PHYSICAL experiment	Work value measured in VIRTUAL experiment
Single Fixed	.49 J	.50 J
	.52 J	.50 J
	.48 J	.50 J
Double Compound	.53 J	.50 J

Physical: Ambiguous
→ Does not promote learning

Virtual Unambiguous
→ Promotes learning

Study 1: Implication of Differential Ambiguity

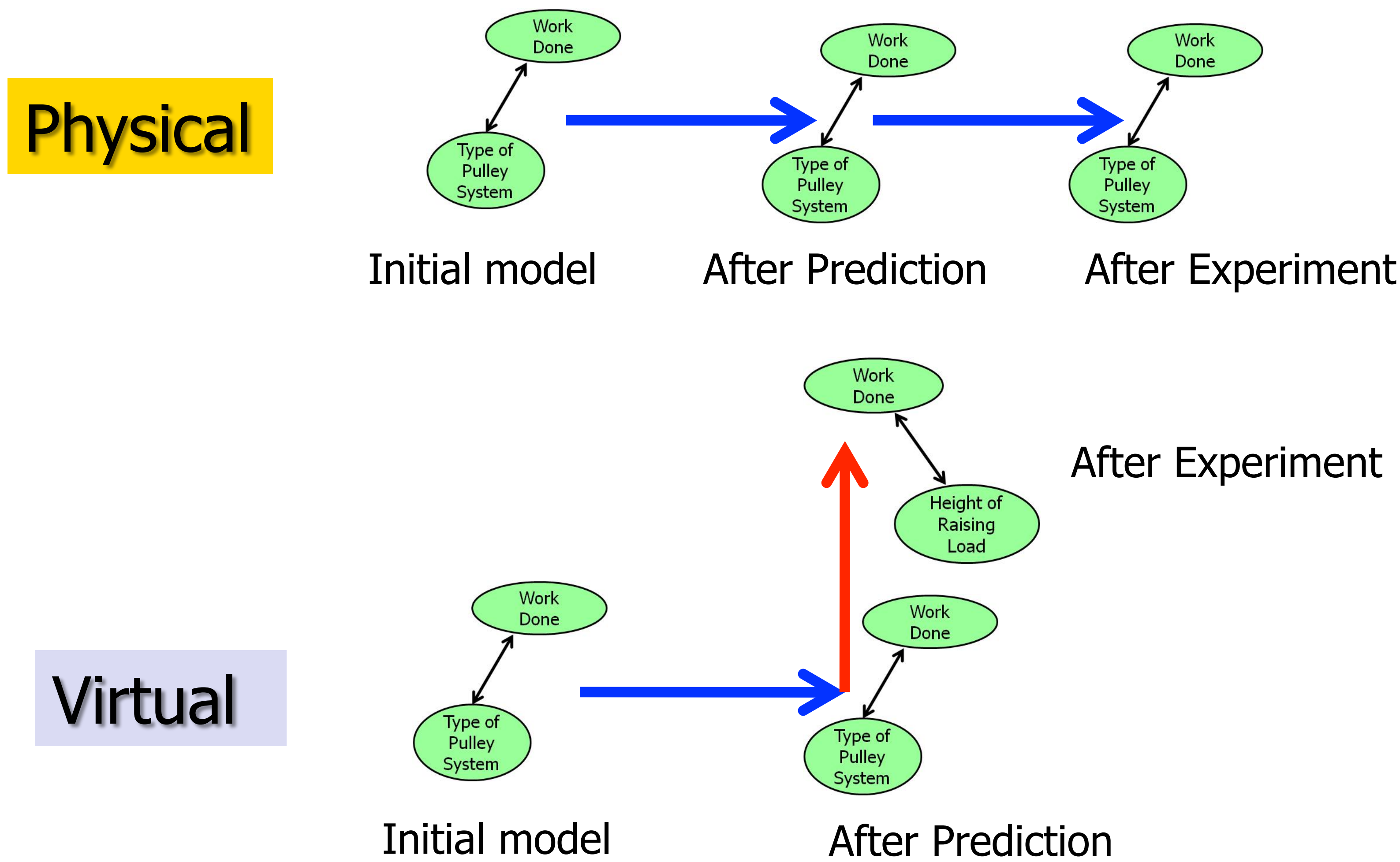
	Physical P	Virtual V
'Force' Data	Unambiguous	Unambiguous
'Work' Data	Ambiguous	Unambiguous

Increasing Unambiguity: Learning

	Virtual V	Physical P
'Force' Cues	Unambiguous	Unambiguous
'Work' Cues	Unambiguous	Ambiguous

Decreasing Unambiguity: No Learning

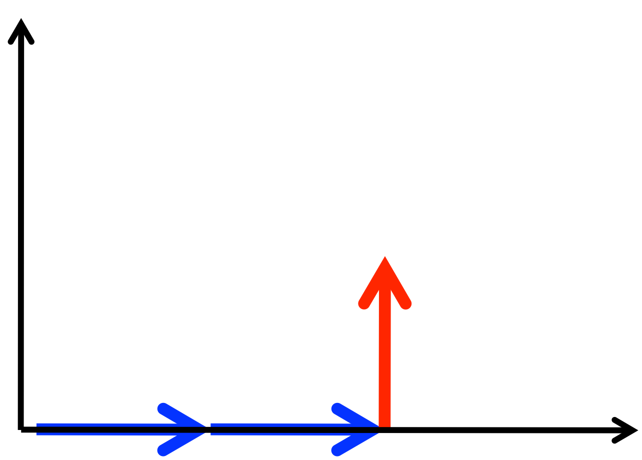
Study 1: Horizontal & Vertical transfer…



Study 1: Horizontal & Vertical transfer…

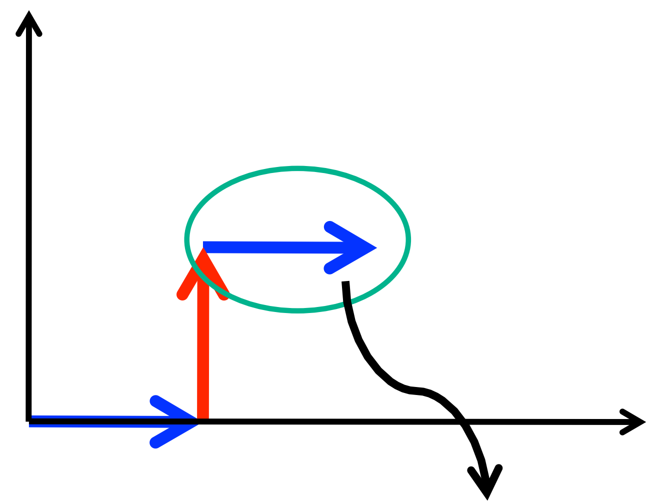
Physical-Virtual Sequence

Physical | Virtual



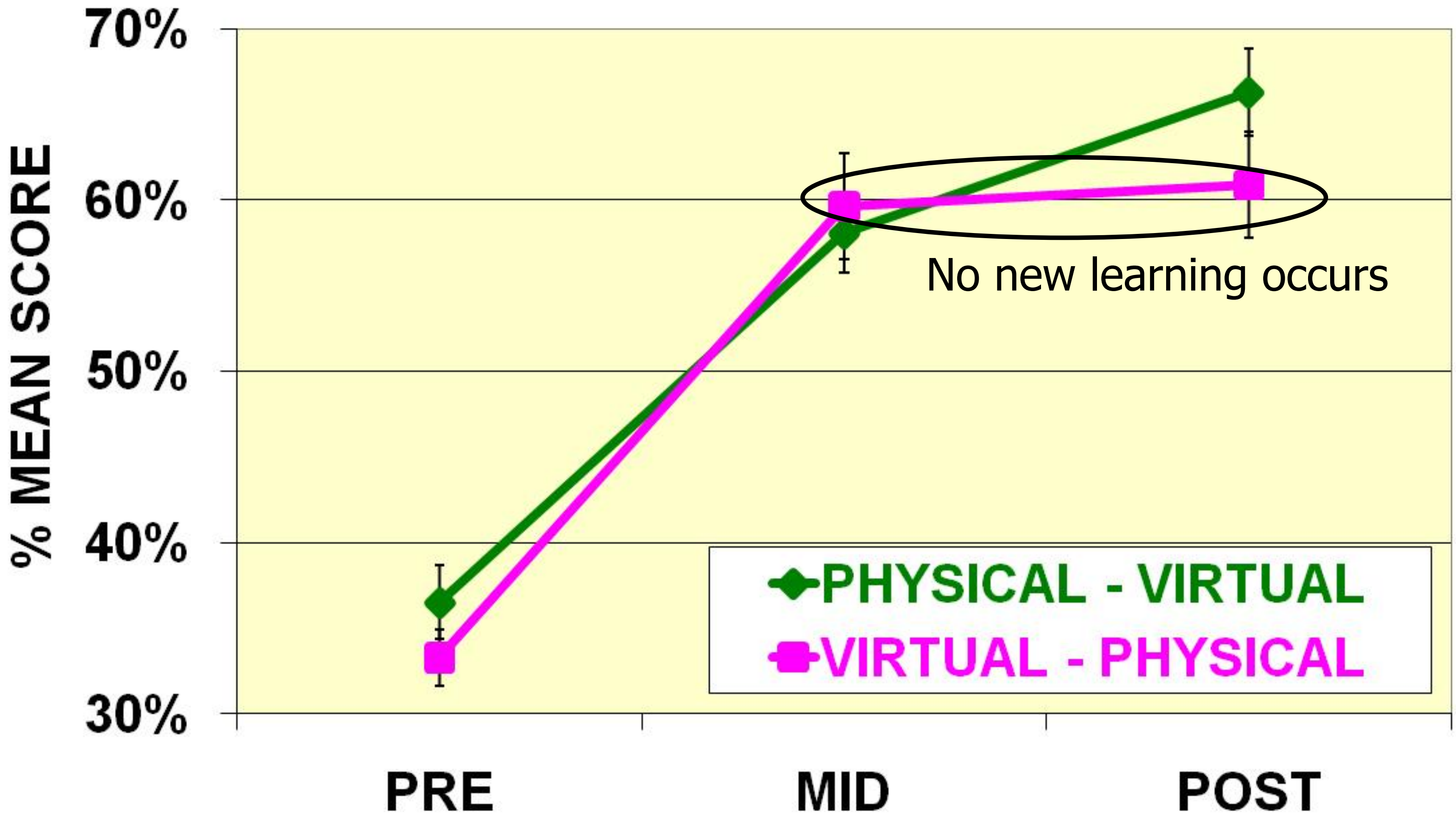
Virtual-Physical Sequence

Virtual | Physical



No new learning occurs in Physical Activity

Study 1: Horizontal & Vertical transfer…



Study 1: Conclusions

When students use both physical & virtual representations…

- Overall, if physical is used first, students continue to learn when virtual is used afterward, but not vice versa

If they don't learn anything more from physical after doing virtual, then why do both, just do virtual?

  - 'Work' : Better learned from virtual rather than physical (Overshadowing, Ambiguity in Data)

RQs For this Talk…

How does the sequence in which learners interact with different representations affect

- learning?
  - Study 1: Learning using Physical vs. Virtual Representations

- problem solving?
  - Study 2: Solving Problems in Numerical vs. Graphical vs. Equational Representations

Study 2: Motivation

Multiple Representations (MRs) useful in solving physics problems

- Several studies addressing the benefits of using MRs in solving physics problems.
- Not as many studies on how students transfer their problem solving skills in physics across different MRs.

Study 2: Research Questions

RQ2.1: What difficulties do students encounter when transferring their problem solving processes across multiple representations?

RQ2.2: How do those difficulties change which the sequence in which these representations are presented?

Study 2: Research Context

- N=20 participants
- Engineering majors
- Enrolled in 1st semester calc-based physics
- Topics: Kinematics, Work-Energy

Study 2: Research Methodology

Data Collection: Teaching/Learning Interviews
(Steffe et al , 2003)

- Four sessions: One after each class exam
- Each session: 60 minutes, video/audio taped
- Three problems per session
- Hints provided when students expressed difficulties

Data Analysis: Phenomenographic coding (Marton, 1986)

- Coded, categorized difficulties expressed by student
- Inter-rater reliability ~ 0.8

Study 2: Research Design

EG Sequence (N=10)

Equation-Graph Sequence

GE Sequence (N=10)

Graph – Equation Sequence

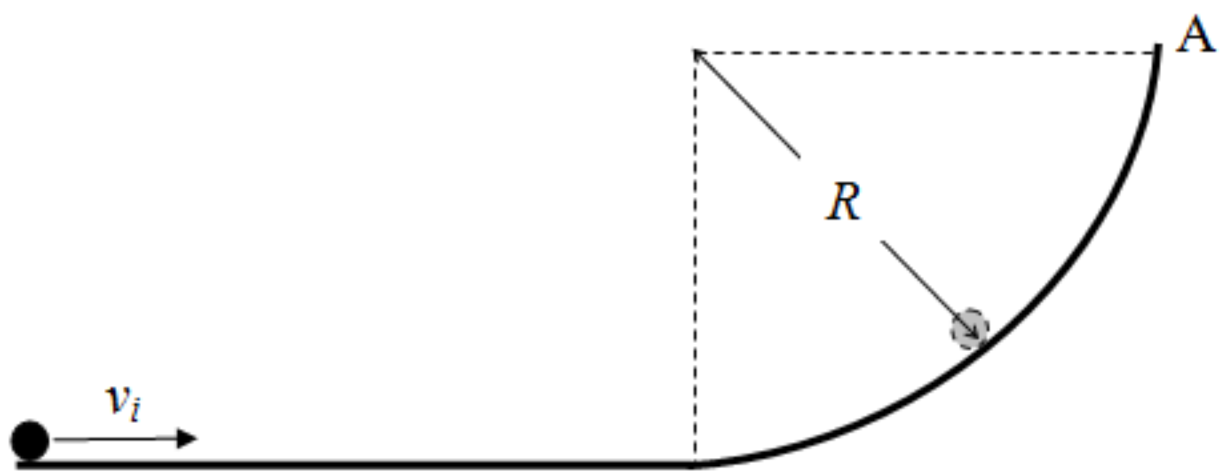
Original (Verbal) Problem

Equational Problem → Graphical Problem

Graphical Problem → Equational Problem

Example: Original Problem (Verbal)

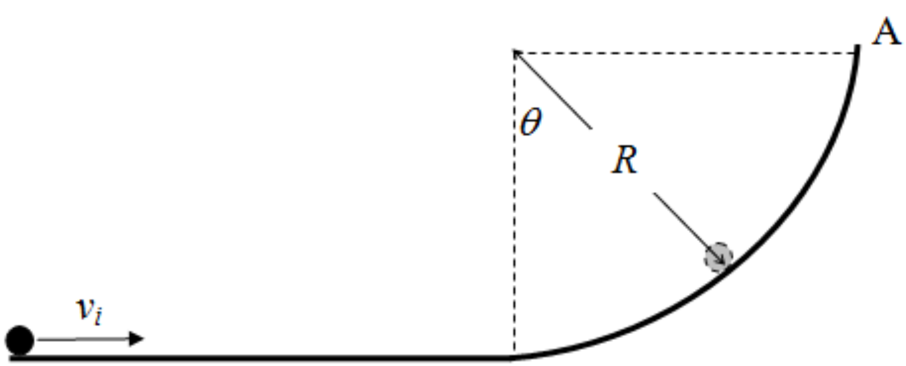
A hoop radius r = 1 cm and mass m = 2 kg is rolling at an initial speed vi of 10 m/s along a track as shown. It hits a curved section (radius R = 2.0 m) and is launched vertically at point A.

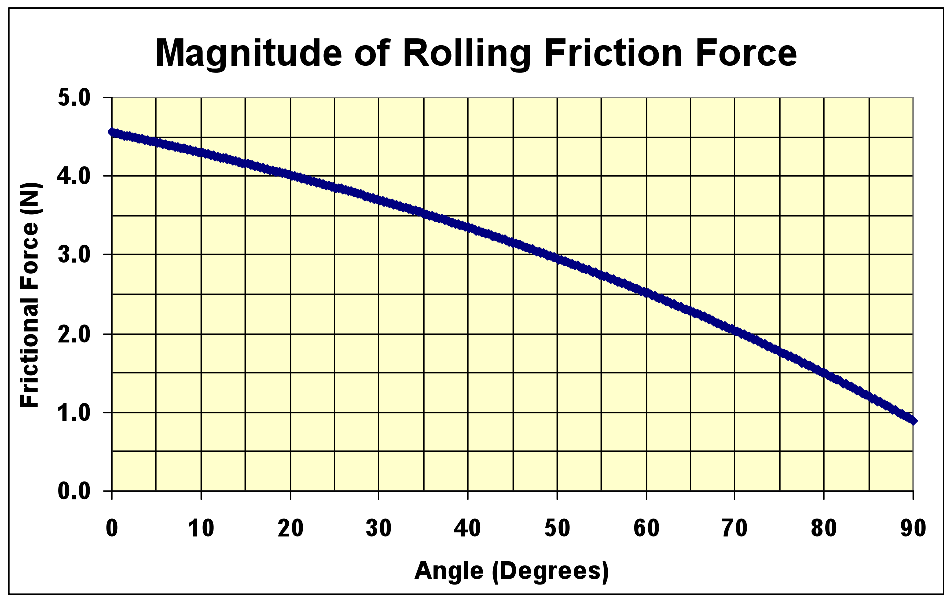


What is the launch speed of the hoop as it leaves the curve at point A?

Example: Graphical Problem

A sphere radius r = 1 cm and mass m = 2 kg is rolling at an initial speed vi of 5 m/s along a track as shown. It hits a curved section (radius R = 1.0 m) and is launched vertically at point A. The rolling friction on the straight section is negligible. The magnitude of the rolling friction force acting on the sphere varies as angle as per the graph shown.

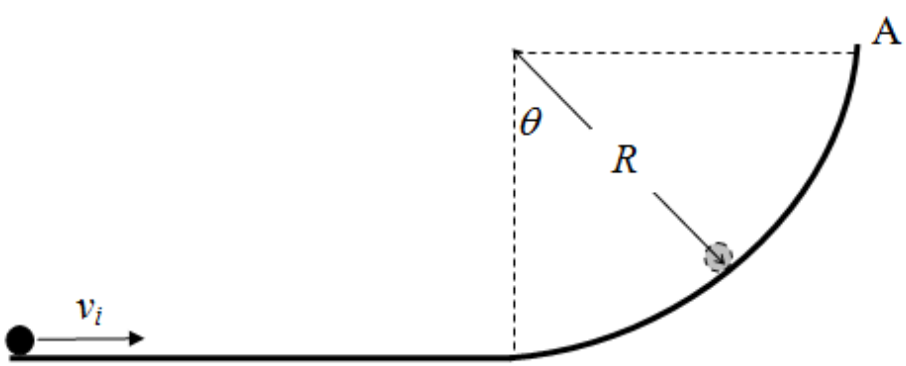




What is the launch speed of the hoop as it leaves the curve at point A?

Example: Equational Problem

A sphere radius r = 1 cm and mass m = 2 kg is rolling at an initial speed vi of 5 m/s along a track as shown. It hits a curved section (radius R = 1.0 m) and is launched vertically at point A. The rolling friction on the straight section is negligible. The magnitude of the rolling friction force acting on the sphere varies as angle (radians) as per the equation shown.



$$F_{roll}(\theta) = -0.7\theta^2 - 1.2\theta + 4.5$$

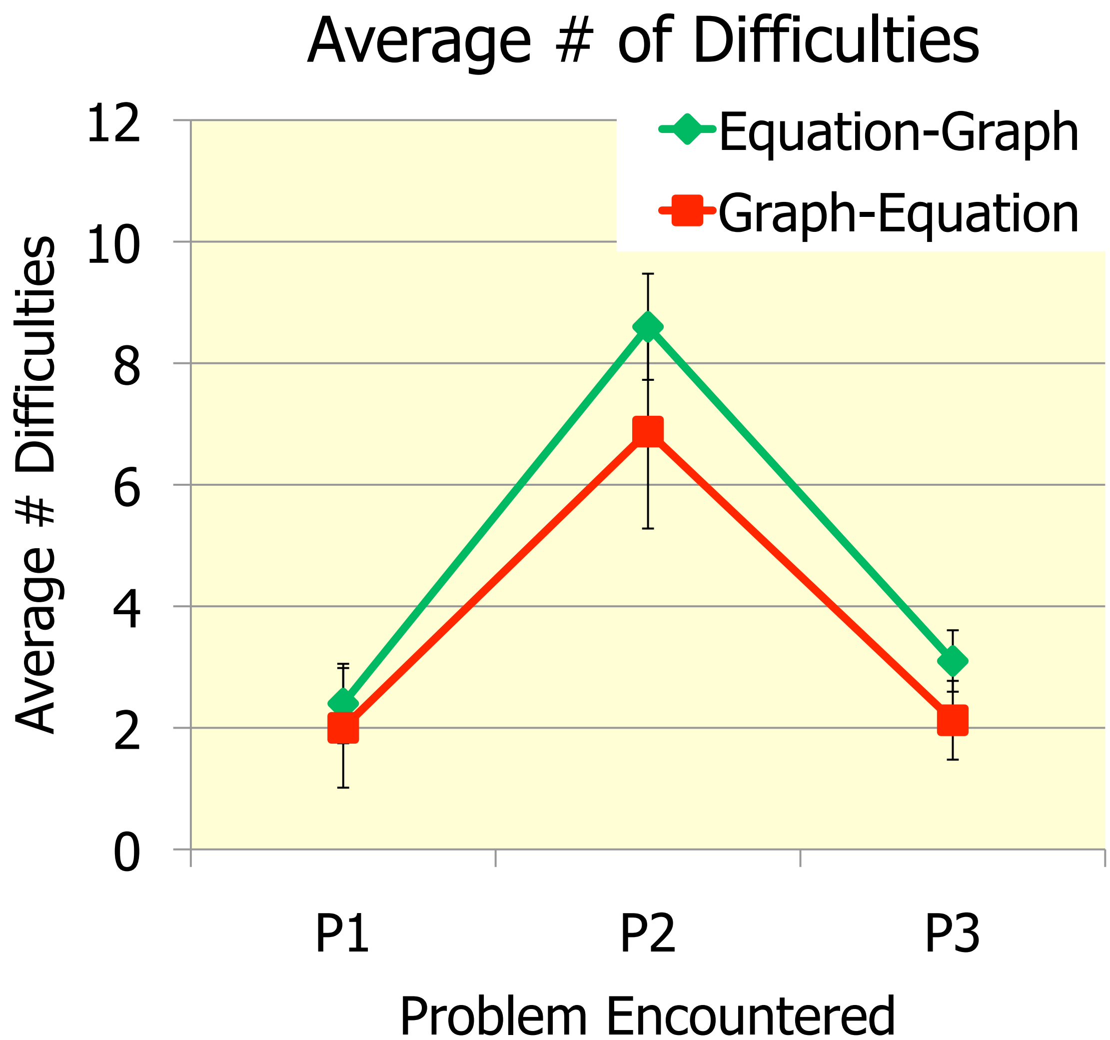
What is the launch speed of the hoop as it leaves the curve at point A?

Study 2: Common Themes

- Case Reuse (Jonassen, 2006)
  - Tried to mimic the previous problems
    - Example: Attempting to find work done by friction by multiplying force with distance.
- Graphical Interpretation
  - Instinctively tried to calculate the slope of graph
  - Several hints to recognize integral is area under graph
- Physical Interpretation of Math Procedures
  - Adequate knowledge of math procedures
  - Inability to apply these procedures in physics problems
  - Hints on reflecting on units of physical quantities effective

Study 2: Results - Sequencing Effect

- Equation-Graph sequence may cause more difficulties to students than the Graph-Equation sequence*



* Not statistically significant

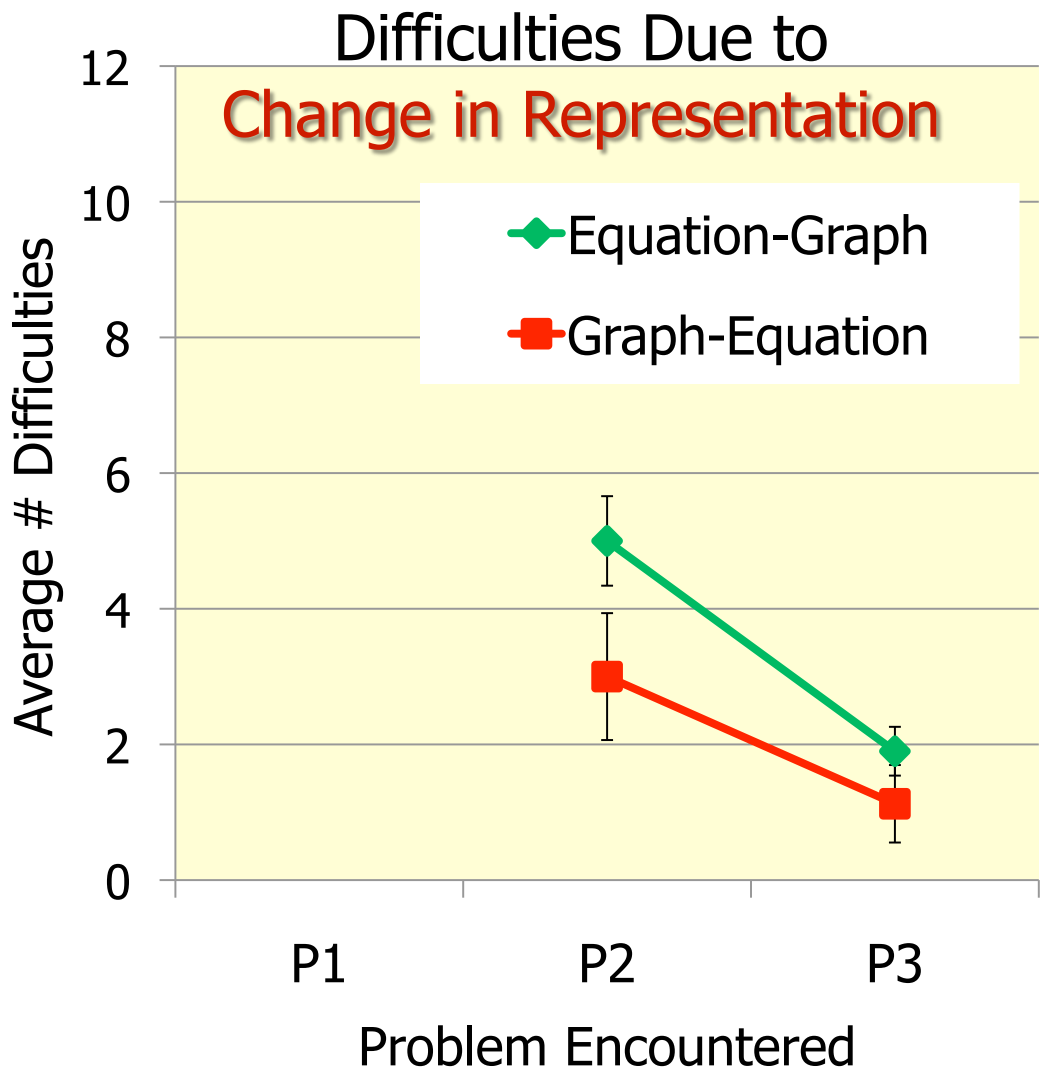
Study 2: Toy Model of Difficulty Contributions

$$D_{Total} = D_{\Delta R} + D_{\Delta C} + D_{\Delta O}$$

- D_{Total} → Total # of Difficulties
- $D_{\Delta R}$ → Difficulties due to Change in Representation
- $D_{\Delta C}$ → Difficulties due to Change in Context
- $D_{\Delta O}$ → Difficulties due to all Other Changes

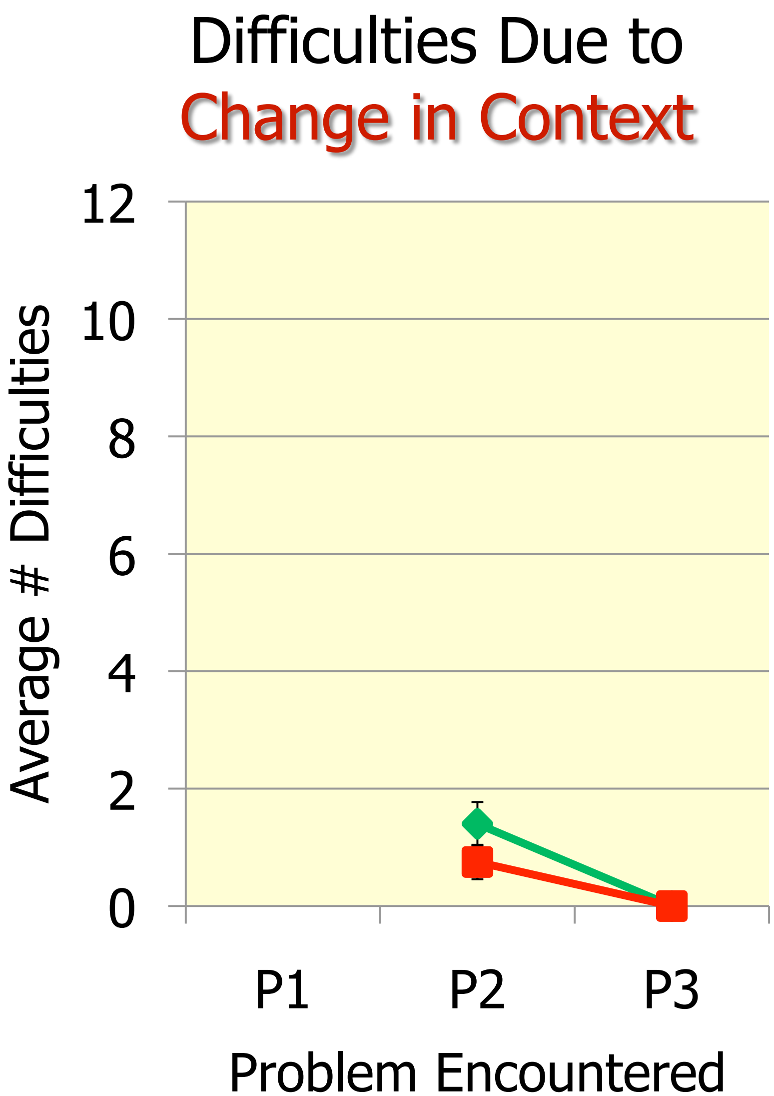
Study 2: Results - Sequencing Effect

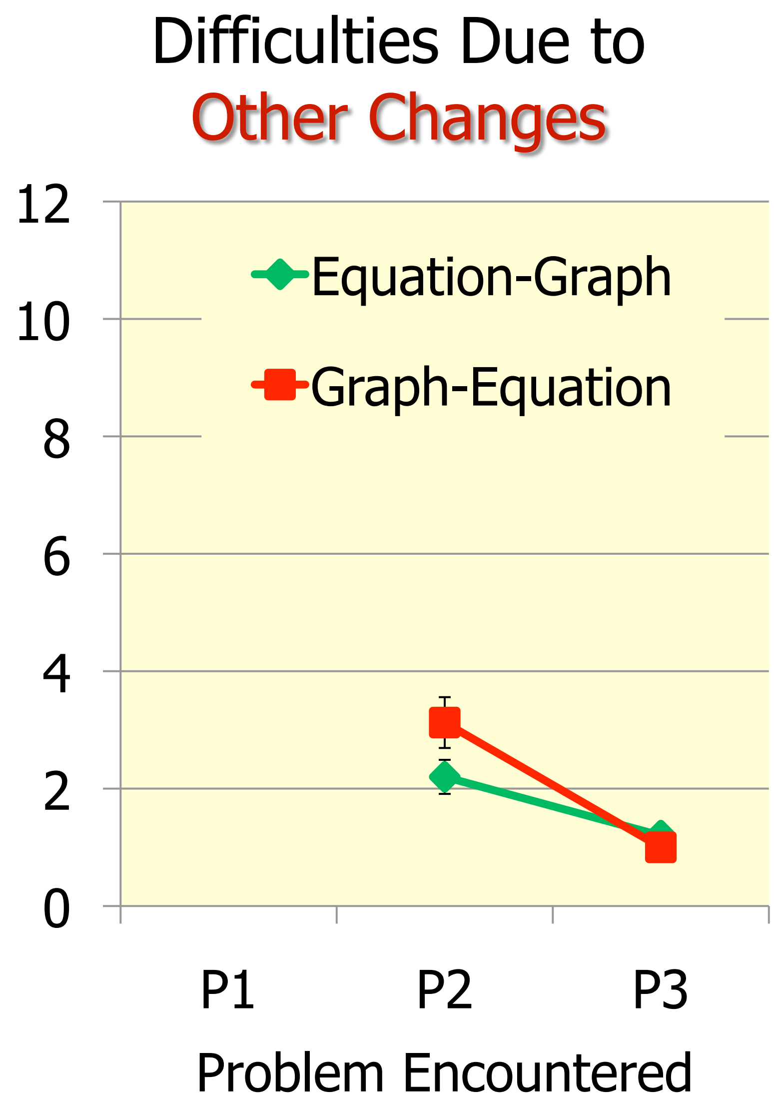
- Most Difficulties are due to change in Representation ($D_{\square R}$)
- Decline in $D_{\square R}$ in going from 2nd problem to 3rd problem regardless of sequence
- $D_{\square R}$ [Verbal → Equation] > $D_{\square R}$ [Verbal → Graph]*



* Not statistically significant

Study 2: Results - Sequencing Effect





Study 2: Conclusions

RQ2.1: What kinds of difficulties do students encounter when solving problems in multiple representations?

- Students had difficulty interpreting physical meaning of mathematical processes.
  - Thus had difficulties solving problems in graphical and functional representations.
- When the context of the problem changed, could not relate the new problem to original problem.
  - Thus had difficulties identifying the principle and physical quantities needed to solve the new problem

Study 2: Conclusions

RQ2.2: How do those difficulties change which the sequence in which these representations are presented?

- Verbal -> Graphical -> Equation sequence has few
  - M to ch ge in co
  - Difficulties due to change in representation are fewer in the G-E sequence compared to E-G sequence.

Why is it easier for students to solve graphical before equational?

Summary

- Different representations offer different salient cues, levels of ambiguity to facilitate and/or overshadow learning of different concepts.
- The sequence in which representations are presented may influence learning & problem solving: Optimal sequencing may be important.

Thank You

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